Name: *Green Solutions*

For full credit, show your work and/or explain your answers.

1. A (fair) die is rolled 3 times. Find the probability that...

(a) (3 pts) ... ***all*** 3 rolls result in ⚅

$$P\left(⚅⚅⚅\right) = \frac{1}{6}\cdot\frac{1}{6}\cdot\frac{1}{6} = \frac{1}{216} \quad (\approx 0.46\%)$$

(b) (3 pts) ... ***at least one*** of 3 the rolls results in ⚅

$$P\left(\text{no ⚅s in 3 rolls}\right) = \frac{5}{6}\cdot\frac{5}{6}\cdot\frac{5}{6} = \frac{125}{216}$$

so

$$P\left(\text{at least one ⚅s in 3 rolls}\right) = 1 - \frac{125}{216} = \frac{91}{216} \quad (\approx 42.13\%).$$

2. (6 pts) One ticket is drawn at random from each of the two boxes

$$A \quad \boxed{\boxed{1}\,\boxed{2}\,\boxed{4}} \qquad B \quad \boxed{\boxed{1}\,\boxed{2}\,\boxed{3}\,\boxed{4}}.$$

Find the probability that the number drawn from box A is smaller than the number drawn from box B.

There are a total of 12 possible pairs of tickets that can be drawn (3 choices from A times 4 choices from B), and they are all equally likely. Of these 12 pairs, only the 5 pairs $(A1, B2)$, $(A1, B3)$, $(A1, B4)$, $(A2, B3)$ and $(A2, B4)$ result in the number from A being smaller than the number from B.

Therefore the probability that the number from A is smaller than the number from B is $\frac{5}{12}$.

3. (3 pts) A box contains 200 tickets: 110 $\boxed{1}$s and 90 $\boxed{0}$s. Tickets are drawn from the box at random with replacement, and you win a dollar if more $\boxed{1}$s are drawn than $\boxed{0}$s. There are two choices:

(i) 20 draws are made from the box.

(ii) 200 draws are made from the box.

Which choice gives a better chance of winning that dollar, (i) or (ii), or do they both give the same chance of winning?

The law of averages says that the more draws that are made from the box, the more likely it is that the proportion of $\boxed{1}$*s in the sample will be close to the proportion of* $\boxed{1}$*s in the box. In this case, this means that with more draws the more likely it is that the proportion of* $\boxed{1}$*s in the sample will be close to 55%.*

To win a dollar, we need the proportion of $\boxed{1}$*s in the sample to be over 50%. This is* ***more*** *likely with a greater number of draws so choice (ii) gives a better chance of winning here.*

4. 400 draws are made at random with replacement from the box $\boxed{\boxed{0}\,\boxed{2}\,\boxed{3}\,\boxed{4}\,\boxed{6}}$.

(a) (3 pts) Find the ***expected value*** of the sum of the draws.

$$\text{Average(box)} = \frac{0+2+3+4+6}{5} = 3 \quad \Longrightarrow \quad \text{EV(sum of draws)} = 400 \times 3 = 1200.$$

(b) (2 pt) What is the probability that the ***observed sum*** of the draws is between 1120 and 1280 — about 43%, about 68% or about 95%? Why?

The possible values for the observed sums have an (approximately) normal distribution with average = $EV(sum)$ and SD = $SE(sum)$. First we compute the SE:

$$\text{SD(box)} = \sqrt{\frac{(0-3)^2+(2-3)^2+(3-3)^2+(4-3)^2+(6-3)^2}{5}} = 2 \quad \Longrightarrow \quad \text{SE(sum of draws)} = \sqrt{400}\times 2 = 40.$$

So $P(1120 \leq \text{observed sum} \leq 1280) \approx 95\%$ because this is the range EV(sum)±2SE(sum).